Section 5

Mathematical skills

Units in calculations

Examiners expect you to use the correct units in calculations

Units

You still describe the speed of a car in miles per hour. The units miles per hour could be written miles/hour or miles hour^{-1} where the superscript '$^{-1}$' is just a way of expressing per something. In science you use the metric system of units, and speed has the unit metres per second, written m s^{-1}. In each case, you can think of per as meaning divided by.

Units can be surprisingly useful

A mile is a unit of distance and an hour a unit of time, so the unit miles per hour reminds you that speed is distance divided by time.

In the same way, if you know the units of density are grams per cubic centimetre, usually written g cm^{-3}, where cm^{-3} means per cubic centimetre, you can remember that density is mass divided by volume.

Multiplying and dividing units

When you are doing calculations, units cancel and multiply just like numbers. This can be a guide to whether you have used the right method.

For example:

The density of a liquid is 0.8 g cm^{-3}. What is the volume of a mass of 1.6 g of it?

Density = mass / volume

So volume = mass / density

Putting in the values and the units:

volume = 1.6 g / 0.8 g cm^{-3}

Cancelling the gs

volume = 2.0 / cm^{-3}

volume = 2.0 cm^{3}

If you had started with the wrong equation, such as

volume = density/mass or

volume = mass × density, you would not have ended up with the correct units for volume.

Units to learn

It is a good idea to learn the units of some basic quantities by heart.

	Unit	Comment
volume	dm^3	1 dm^3 is 1 litre, L, which is 1000 cm^3
concentration	mol dm^{-3}	
pressure	Pascals, Pa = N m^{-2}	N m^{-2} are newtons per square metre
enthalpy	kJ mol^{-1}	kJ is kilojoule. Occasionally J mol^{-1} is used.
entropy	JK^{-1} mol^{-1}	joules per kelvin per mole

Standard form

This is a way of writing very large and very small numbers in a way that makes calculations and comparisons easier.

The number is written as number multiplied by ten raised to a power. The decimal point is put to the right of the first digit of the number.

For example:

22 000 is written 2.2×10^4.

0.000 002 2 is written 2.2×10^{-6}.

How to work out the power to which ten must be raised

Count the number of places you must move the decimal point in order to have one digit before the decimal point.

For example:

$0.00051 = 5.1 \times 10^{-4}$

$51000 = 5.1 \times 10^{4}$

Moving the decimal point to the right gives a negative index (numbers less than 1), and to the left a positive index (numbers greater than one).

(The number 1 itself is 10^0, so the numbers 1–9 are followed by $\times 10^0$ when written in standard form. Can you see why?)

Multiplying and dividing

When *multiplying* numbers expressed in this way, *add* the powers (called indices) and when *dividing*, *subtract* them.

Worked examples

Calculate

a $2 \times 10^5 \times 4 \times 10^6$

b $\dfrac{8 \times 10^3}{4 \times 10^2}$

c $\dfrac{5 \times 10^8}{2 \times 10^{-6}}$

Answer

a $2 \times 10^5 \times 4 \times 10^6$

Multiply $2 \times 4 = 8$. Add the indices to give 10^{11}

Answer = 8×10^{11}

b $\dfrac{8 \times 10^3}{4 \times 10^2}$

Divide 8 by 4 = 2. Subtract the indices to give 10^1

Answer = $2 \times 10^1 = 20$

c $\dfrac{5 \times 10^8}{2 \times 10^{-6}}$

Divide 5 by 2 = 2.5. Subtract the indices $(8 - (-6))$ to give 10^{14}

Answer = 2.5×10^{14}

A handy hint for non-mathematicians

Non-mathematicians sometimes lose confidence when using small numbers such as 0.002. If you are not sure whether to multiply or divide, then do a similar calculation with numbers that you are happy with, because the rule will be the same.

Example:

How many moles of water in 0.000 1 g? A mole of water has a mass of 18 g.

Do you divide 18 by 0.000 1 or 0.000 1 by 18?

If you have any doubts about how to do this, then in your head change 0.000 1 g into a more familiar number such as 100 g.

How many moles of water in 100 g? A mole of water has a mass of 18 g.

Now you can see that you must divide 100 by 18. So in the same way you must divide 0.000 1 by 18 in the original problem.

$\dfrac{0.0001}{18} = 5.6 \times 10^{-6}$

Prefixes and suffixes

In chemistry you will often encounter very large numbers (such as the number of atoms in a mole) or very small numbers (such as the size of an atom).

Prefixes and suffixes are often used with units to help express these numbers. You will come across the following which multiply the number by a factor of 10^n. The red ones are the ones you are most likely to use.

Prefix	Conversion Factor	Symbol
pico	10^{-12}	p
nano	10^{-9}	n
micro	10^{-6}	μ
milli	10^{-3}	m
centi	10^{-2}	c
deci	10^{-1}	d
kilo	10^{3}	k
mega	10^{6}	M

So 5400 g = 5.4×10^3 g = 5.4 kg

Converting to base units

If you want to convert a number expressed with a prefix to one expressed in the base unit, multiply by the conversion factor. If you have a very small or very large number (and have to handle several zeros) the easiest way is to first convert the number to standard form.

Worked example

Convert a) 2 cm and b) 100 000 000 mm to metres

a $2\,cm = 2 \times 10^{-2}\,m = 0.02\,m$

b $100\,000\,000\,mm = 1 \times 10^{8}\,mm$

$= 1 \times 10^{8} \times 10^{-3}\,m = 1 \times 10^{5}\,m$

conversion factor

Base units

The SI system is founded on base units. The ones you will meet in chemistry are:

Unit	Symbol	Used for
metre	m	length
kilogram	kg	mass
second	s	time
ampere (amp)	A	electric current
kelvin	K	temperature
mole	mol	amount of substance

Handling data

Sorting out significant figures

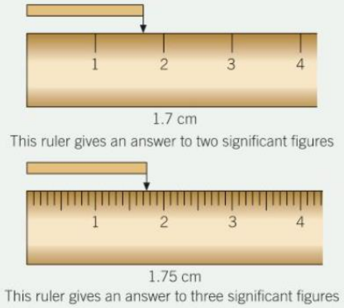
Many of the numbers used in chemistry are measurements – for example, the volume of a liquid, the mass of a solid, the temperature of a reaction vessel – and no measurement can be exact. When you make a measurement, you can indicate how uncertain it is by the way you write it. For example a length of 5.0 cm means that you have used a measuring device capable of reading to 0.1 cm, a value of 5.00 cm means that you have measured to the nearest 0.01 cm and so on. So the numbers 5, 5.0 and 5.00 are different, you say they have different numbers of *significant figures*.

What exactly is a significant figure?

In a number that has been found or worked out from a measurement, the significant figures are all the digits known for certain, *plus the first uncertain one* (which may be a zero). The last digit is the uncertain one and is at the limit of the apparatus used for measuring it (Figure 1).

▲ **Figure 1** *A number with four significant figures*

For example if you say a substance has a mass of 4.56 grams it means that you are certain about the 4 and the 5 but not the 6 as you are approaching the limit of accuracy of our measuring device (you will have seen the last figure on a top pan balance fluctuate). The number 4.56 has three significant figures (s.f.).



▲ **Figure 2** *Rulers with different precision*

When a number contains zeros, the rules for working out the number of significant figures are given below.

- Zeros between digits are significant.
- Zeros to the left of the first non-zero digit are not significant (even when there is a decimal point in the number).
- When a number with a decimal point ends in zeros to the right of the decimal point, these zeros are significant.
- When a number with no decimal point ends in several zeros, these zeros may or may not be significant. The number of significant figures should ideally be stated. For example 20 000 (to 3 s.f.) means that the number has been measured to the nearest 100 but 20 000 (to 4 s.f.) means that the number has been measured to the nearest 10.

The following examples should help you to work out the number of significant figures in your data.

Worked examples

What is the number of significant figures in each of the following?

a 11.23

Answer

4 s.f. all non-zero digits are significant.

b 1100

Answer

2 s.f. (but it could be 2, 3, or 4 significant figures). The number has no decimal point so the zeros may or may not be significant. With numbers with zeros at the end it is best to state the number of significant figures.

c 1100.0

Answer

5 s.f. the decimal point implies a different accuracy of measurement to example (b).

d 0.025

Answer

2 s.f. zeros to the left of the decimal point only fix the position of the decimal point. They are not significant.

Question

1 How many significant figures?

a 40 000

b 1.030

c 0.22

d 22.00

Using significant figures in answers

When doing a calculation, it is important that you don't just copy down the display of your calculator, because this may have a far greater number of significant figures than the data in the question justifies. Your answer cannot be more certain than the least certain of the information that you used to calculate it. So your answer should contain the same number of significant figures as the measurement that has the smallest number of them.

Worked example

81.0 g (3 s.f.) of iron has a volume of 10.16 cm^3 (4 s.f.). What is its density?

Answer

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{81.0\ \text{g}}{10.16\ \text{cm}^3}$$

$= 7.972\,440\,94\ \text{g cm}^{-3}$ (this number has 9 s.f.)

Since our least certain measurement was to 3 s.f., our answer should have 3 s.f., ie 7.97 g cm^{-3}

If our answer had been 7.976 440 94, you would have rounded it ip to 7.98 because the fourth significant figure (6) is five or greater.

The other point to be careful about is *when* to round up. This is best left to the very end of the calculation. Don't round up as you go along as this could make a difference to your final answer.

Decimal places and significant figures

The apparatus you use in the laboratory usually reads to a given number of decimal places (for example hundredths or thousandths of a gram). For example, the top pan balances in most schools and colleges usually weigh to 0.01 g which is to two decimal places.

The number of significant figures of a measurement obtained by using the balance depends on the mass you are finding. A mass of 10.38 g has 4 s.f. but a mass of 0.08 has only 1 s.f. Check this with the rules above.

Hint

Calculator displays usually show numbers in standard form in a particular way. For example 2.6×10^{-4} may appear as 2.6 – 04, a shorthand form which is not acceptable as a way of writing and answer. It is an error that examiners often complain about.

Algebra

Equations

You can write an equation if you can show a connection between sets of measurements (variables).

For example, at a *fixed* volume, if you double the temperature (in Kelvin) of a gas, the pressure doubles too.

Mathematically speaking, the pressure P is directly proportional to the temperature T.

$P \propto T$

The symbol $\propto$ means is proportional to.

This is shown in the data in Table 1

▼ **Table 1**

Temperature/K	Pressure/Pa
100	1000
150	1500
200	2000
250	2500

This also means that the pressure, P, is equal to some constant, k, multiplied by the temperature:

$P = kT$

In this case, the constant is 10 and if you multiply the temperature in K by 10, you get the pressure in Pa.

Pressure and volume of a gas also vary. At constant temperature, as the pressure of a gas goes up, its volume goes down. More precisely, if you double the pressure, you halve the volume. Mathematically speaking, volume *V*, is *inversely* proportional to pressure *P*.

$$V \propto \frac{1}{P}$$

So $V = k \times \frac{1}{P}$

Or simply $V = \frac{k}{P}$

This is shown by the data in Table 2.

In this case the constant is 24. If you multiply $\frac{1}{P}$ by 24, you get the volume.

▼ **Table 2**

Pressure/Pa	Volume/l	1/P = 1/Pa
1	24	1.00
2	12	0.50
3	8	0.33
4	6	0.25

Mathematical symbols	
Symbol	**Meaning**
$\rightleftharpoons$	equilibrium
$<$	less than
$\ll$	much less than
$>$	greater than
$\gg$	much greater than
$\sim$	approximately equal to
$\propto$	proportional to

Question

2a If two variables, x and y, are directly proportional to each other, what happens to one if you quadruple the other?

b Write an expression that means x is inversely proportional to y.

c What happens to the volume of a gas if you triple the pressure at constant temperature?

Handling equations

Changing the subject of an equation

If you can confidently do the next exercise, go straight to the section Substituting into equations and try that. Otherwise work through Rearranging equations.

Question

3 The equation that connects the pressure *P*, volume *V* and temperature *T* of a mole of gas is $PV = RT$

Where *P*, *V*, and *T* are variables and *R* is a constant called the gas constant.

Rearrange the equation to find:

a P in terms of V, R and T

b V in terms of P, R and T

c T in terms of P, V and R

d R in terms of P, V and T

Rearranging equations

Start with a simple relation because the rules are the same however complicated the equation.

$$a = \frac{b}{c}$$

where $b = 10$ and $c = 5$,

It is easy to see that substituting these values into the expression

$$a = \frac{10}{5} = 2.$$

But what do you do if you need to find *b* or *c* from this equation?

you need to rearrange the equation so that *b* (or *c*) appears on its own on the left hand side of the equation like this

$b = ?$

$a = \frac{b}{c}$ means $a = b \div c$

Step 1: Multiply both sides of the equation by *c*, because *b* is *divided* by *c*, so to get *b* on it own you must *multiply* by *c*.

Remember that to keep an equation valid whatever you do to one side you must do to the other – think of it as a see-saw, with the = sign as the pivot.

So now $c \times a = \frac{b \times c}{c}$

usually written $ca = \frac{bc}{c}$

Now cancel the *c*'s on the right since *b* is being both multiplied and divided by *c*

Which leaves $c \times a = b$

Or $b = c \times a$ usually written $b = ca$

You can now rearrange this equation in the same way to find *c*.

Step 2: $b = c \times a$. Divide both sides by *a*

$$\frac{b}{a} = \frac{cax}{a}$$

Now cancel the *a*'s on the right

So $\frac{b}{a} = c$

Notice that because *c* started on the bottom, a two-step process was necessary. You found as expression for *b* first and then found one for *c*.

Question

4 Find the variable in brackets in terms of the others.

a $p = \frac{q}{r}$ (q)

b $n = mt$ (m)

c $g = \frac{fe}{h}$ (h)

d $\frac{pr}{e} = s$ (r)

Substituting into equations

When you are asked to substitute numerical values into an equation, it is essential that you carry out the mathematical operations in the right order.

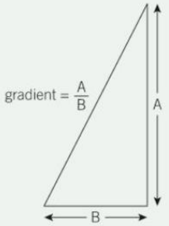
A useful aid to remembering the order is the word **BIDMAS**:

Brackets
Indices
Division
Multiplication
Addition
Subtraction

Graphs

The graph in Figures 3 and 4 show the rate of reaction between hydrochloric acid and magnesium to produce hydrogen gas.

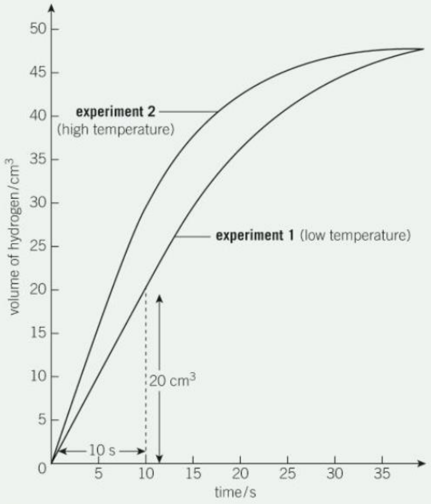
The gradient of a straight line section of a graph is found by dividing the length of the line A (vertical) by the length of line B (horizontal).



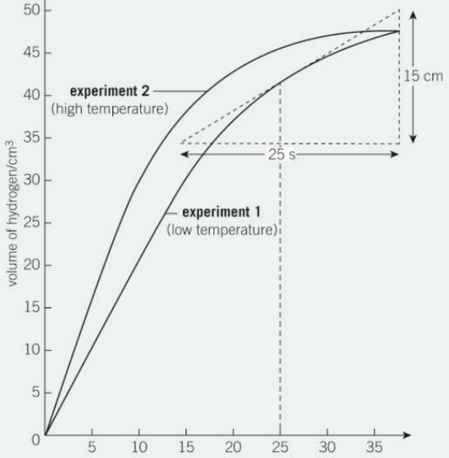
In the case of experiment 1 above this tells us the rate at which hydrogen is produced, between 0 ands 10 seconds. It will have units:

Rate = $20\,cm^3/10\,s = 2\,cm^3/s$

or $2\,cm^3s^{-1}$



▲ **Figure 3** *Reaction rate graph*



▲ **Figure 4** *Reaction rate graph*

The steeper the line is, the greater the rate.

Question

5 Find the rate for experiment 2 over the first 10 seconds.

Tangents

When you get to the part of the graph where the line starts to curve, the best you can do is to take a tangent at the point you are investigating and find the gradient of the tangent.

A tangent is a line drawn so that it just touches the graph line at one particular point.

The rate in experiment 1, after 25 seconds is $\frac{15\,\text{cm}^3}{25\,\text{s}} = 0.6\ \text{cm}^3\,\text{s}^{-1}$

Questions

6 Find the rate after 30 s in experiment 2.

Logarithms

A logarithm, or log for short, is a mathematical function – the log of a number represents the power to which a base number (often ten) has to be raised to give the number. This is easy to do for numbers that are multiples of ten such as 100 or 10000. 100 is 10^2, so the log to the base ten (written $\log_{10}$ or just log) of 100 is 2 and $\log_{10}$ of 10,000 is 4. Logs can also be negative numbers. The log of $\frac{1}{1000}$ is −3 as $\frac{1}{1000}$ is 10^{-3}. With other numbers you must use a calculator to find the log. Log_{10} 72.33 is 1.8593.

Make sure that you are confident using your calculator to find logs.

Question

7 What is the $\log_{10}$ of:

- **a** 1000
- **b** $\frac{1}{100}$
- **c** 0.0001
- **d** 48.2
- **e** 0.037

You will need to use a calculator for the last two you can go back from the log to the original number by using the antilog, or inverse log function, of the calculator. Log (10^{27}) is 27 (which doesn't need a calculator to work out) and log (21379.6) is 4.33.

Make sure that you are confident to use your calculator to find antilogs (inverse logs).

Question

8 What is the antilog (inverse log) of:

- **a** 3
- **b** −2
- **c** 14
- **d** 8.2
- **e** 0.37

You will need to use a calculator for the last two. The log function turns very large or very small numbers into more manageable numbers without losing the original number (which you can recover using the antilog function). So $\log_{10} 6\times10^{23} = 23.778$ and $\log_{10} 1.6\times10^{-19} = -18.79$.

This can be very useful in plotting graphs as it can allow numbers with a wide range in magnitudes to be fitted onto a reasonable size of graph. For example, the successive ionisation energies of sodium range from 496 to 159079 whereas their logs range from just 2.695 to 5.201.

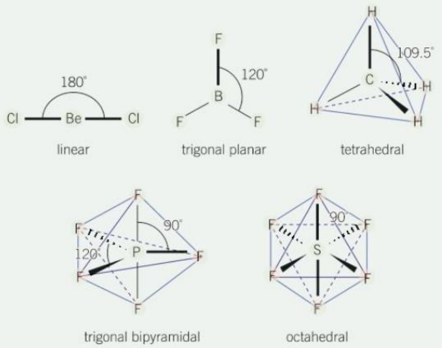
Another important use of logs in chemistry is the pH scale, which measures acidity, and depends on the concentration of hydrogen ions (H^+) in a solution. This can vary from around 5 mol dm^{-3} to around 5×10^{-15} mol dm^{-3} – an enormous range. If you use a log scale, this becomes 0.6989 to −14.301 – a much more manageable range.

When multiplying numbers you add their logs and when dividing you subtract them. This is easy to see with numbers that are multiples of ten.

100 × 10000 = 1000000, that is, $10^2 \times 10^4 = 10^6$ and $\frac{10^4}{10^2} = 10^2$, but the same rules apply for more awkward numbers.

Geometry and trigonometry

Simple molecules adopt a variety of shapes. The most important of these are shown below with the relevant angles. When drawing representations of three dimensional shapes, the convention is to show bonds coming out of the paper as wedges which get thicker as they come towards you. Bonds going into the paper are usually drawn as dotted lines or reverse wedges.



▲ **Figure 5** *Three-dimensional drawings of molecular shapes*

The three-dimensional shapes are based on geometrical solid figures as shown